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Spectrum Analyzer Noise De-Embedding for Accurate Measurements

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This article explains why the most precise measurements must consider the internal noise of the instrumentation and descibes the method for finding the proper correction factor

Spectrum analyzers are widely used for RF signals measurements. Due to the internal noise of the analyzer's measurement circuitry, the signal power level shown is greater than the actual value. Accurate power level measurements of RF signals require the addition of a correction factor d to remove the effect of the test equipment's non-ideal noise figure.

This article first addresses measurement of the noise figure of RF devices, then explains noise de-embedding techniques that are used to determine and use the correction factor for accurate measurement of power levels.

Spectrum Analyzer Noise Floor

Like all RF devices, a spectrum analyzer has internally generated thermal noise which limits the lowest RF power level that can be measured and displayed on the screen. This level is known as Minimum Discernible Signal (MDS) and is defined by:

$$\text{MDS} = 10\log(ktB) + NF \qquad (1)$$

where:

k is the Boltzmann entropy constant [1], $k = 1.38066 \cdot 10^{-23} \text{JK}^{-1}$ or $1.38066 \cdot 10^{-23}\ \text{W} \cdot \text{S} \cdot \text{K}^{-1}$. This number represents the conversion factor between two forms of energy. It gives the average mechanical energy per particle which can be coupled out electrically per K. In RF engineering another often used number is $1.38066 \cdot 10^{-20}\ \text{mW} \cdot \text{S} \cdot \text{K}^{-1}$;

T is temperature in K;

B is noise bandwidth of the measurement in Hz;

NF is spectrum analyzer noise figure.

For every 1 Hz of noise bandwidth at room temperature, T_0 = 290K, we have a noise power, $P_{1\text{Hz}}$:

$$P_{1\text{Hz}} = k \cdot T_0 \cdot 1 = 1.38066 \cdot 10^{-23} \cdot 290 = 4.003914 \cdot 10^{-21}\frac{\text{W}}{\text{Hz}} = 173.975\frac{\text{dBm}}{\text{Hz}} \qquad (2)$$

For a system with a noise bandwidth of B, the noise power delivered by a room temperature noise source is

$$N_0 = 173.975\frac{\text{dBm}}{\text{Hz}} + 10\log(B) \qquad (3)$$

which corresponds to –143.975 dBm for 1 kHz, –133.975 dBm for 10 kHz and –123.975 dBm for 100 kHz noise bandwidth. Consider a typical noise figure for spectrum analyzers of about 30 dB. This yields noise floor (MDS) levels on the order of –114 dBm for 1 kHz, –104 dBm for 10 kHz and –94 dBm for 100 kHz noise bandwidth. As the measured signal approaches the MDS level, more internal noise affects the measured signal level and the more important the correction factor d will be. The rule of thumb is to keep measurement circuit noise floor at least 10 dB below signal noise floor. In such case measured signal level will be less than 0.46 dB larger of actual value.

D	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	–6.868	–6.502	–6.172	–5.872	–5.598	–5.345	–5.112	–4.896	–4.694	–4.506
2	–4.329	–4.163	–4.007	–3.860	–3.721	–3.589	–3.463	–3.344	–3.231	–3.123
3	–3.021	–2.922	–2.829	–2.739	–2.653	–2.570	–2.491	–2.415	–2.342	–2.272
4	–2.205	–2.140	–2.077	–2.017	–1.959	–1.903	–1.849	–1.797	–1.747	–1.698
5	–1.651	–1.605	–1.561	–1.519	–1.478	–1.438	–1.399	–1.362	–1.325	–1.290
6	–1.256	–1.223	–1.191	–1.160	–1.130	–1.101	–1.072	–1.045	–1.018	–0.992
7	–0.967	–0.942	–0.918	–0.895	–0.872	–0.850	–0.829	–0.808	–0.788	–0.768
8	–0.749	–0.731	–0.713	–0.695	–0.678	–0.661	–0.645	–0.629	–0.614	–0.599
9	–0.584	–0.570	–0.556	–0.543	–0.530	–0.517	–0.504	–0.492	–0.480	–0.469
10	–0.458	–0.447	–0.436	–0.425	–0.415	–0.405	–0.396	–0.386	–0.377	–0.368
11	–0.359	–0.351	–0.343	–0.335	–0.327	–0.319	–0.311	–0.304	–0.297	–0.290
12	–0.283	–0.276	–0.270	–0.264	–0.257	–0.251	–0.245	–0.240	–0.234	–0.229
13	–0.223	–0.218	–0.213	–0.208	–0.203	–0.198	–0.194	–0.189	–0.185	–0.181
14	–0.176	–0.172	–0.168	–0.164	–0.161	–0.157	–0.153	–0.150	–0.146	–0.143
15	–0.140	–0.136	–0.133	–0.130	–0.127	–0.124	–0.121	–0.118	–0.116	–0.113
16	–0.110	–0.108	–0.105	–0.103	–0.101	–0.098	–0.096	–0.094	–0.092	–0.090
17	–0.088	–0.086	–0.084	–0.082	–0.080	–0.078	–0.076	–0.074	–0.073	–0.071
18	–0.069	–0.068	–0.066	–0.065	–0.063	–0.062	–0.060	–0.059	–0.058	–0.056
19	–0.055	–0.054	–0.053	–0.051	–0.050	–0.049	–0.048	–0.047	–0.046	–0.045

Table 1 · Correction factors for spectrum analyzer noise de-embedding.

Determining of the Correction Factor d

RF signals measured with a spectrum analyzer are usually uncorrelated with respect to instrument noise. The latter combines in a power addition with measuring signal to provide a larger measured value than actual. Smaller differences between total measurement setup noise and internal instrument noise result in larger differences between measured and actual signal level.

In order to determine the value of the correction factor d, it is necessary to measure the difference D in dB between total and internal noise floor levels. Then calculate the correction factor from the following equation:

$$\delta = 10\log\left(10^{\frac{\Delta}{10}} - 1\right) - \Delta \qquad (4)$$

Values of the desired correction factor can also be found from Table 1.

Measuring the difference in noise floors is shown in Figure 1. First we measure the total noise floor of whole measurement circuit N_t with the input cable connected to measurement circuit (upper noise trace on Figure 1). Then we disconnect the input cable and measure the internal noise floor of the measurement circuit N_i (lower noise trace on Figure 1). To increase the accuracy of the noise floor measurements, averaging can be used. After N_t and N_i values are obtained, the difference, D, can easily be calculated by D $= N_t - N_i$.

The corrected value of signal power level P_{corr} can be determined from measured level P_{meas} and correction factor d as:

$$P_{corr} = P_{meas} + \mathrm{d} \qquad (5)$$

Noise De-Embedding Technique for *Y*-Method of Noise Figure Measurement

There is a well known *Y*-method of noise figure measurement that is based on the injection of a known amount of noise into a system and observing the system output behavior. A noise source should be capable of producing a significant noise power exceeding that of the device being measured. Figure 2 shows a simplified

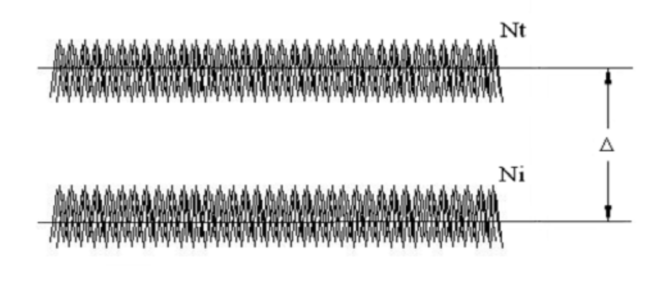

Figure 1 · Illustration of D, the difference between N_t (total) and N_i (internal) noise floor levels.

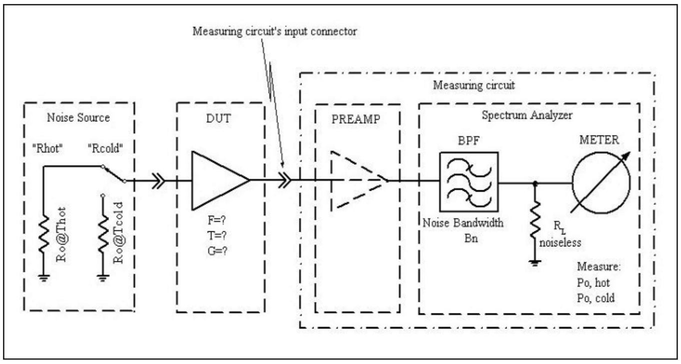

Figure 2 · *Y*-method noise measurement system.

schematic diagram of the noise measurement setup.

The noise source has two resistors, “R_{hot}” and “R_{cold}.” Both have a value R_0, which is the characteristic impedance of the system. These resistors are at different noise temperatures: R_{hot} is at T_{hot} (usually about 10,000K), and R_{cold} is at T_{cold} (usually about room temperature, 290K). We know T_{hot} and T_{cold} with a high degree of accuracy. During the measurement, we alternately connect first the hot resistor to the device under test (DUT), then connect the cold resistor to it. The output of the DUT goes to a band-pass filter that sets the noise bandwidth of the measurement. The actual bandwidth does not matter because it cancels out in the equations. What is important is to keep bandwidth constant and not to switch the filter during the noise measurement.

The meter measures the noise energy at the band-pass filter output in a load resistor R_L. When R_{hot} is connected to the DUT meter it measures $P_{0,hot}$, and when R_{cold} is connected $P_{0,cold}$ is measured. The ratio of these two values provides us with the Y value

$$Y = 10^{\left(\frac{P_{0,hot} - P_{0,cold}}{10}\right)} \quad (6)$$

Here $P_{0,hot}$ and $P_{0,cold}$ power levels are in decibels and Y is in linear term.

To increase the accuracy of $P_{0,hot}$ and $P_{0,cold}$ measurements the above described technique may be used. We need to determine the correction factors δ_h and δ_c for both measurements and then substitute them into equation

$$Y = 10^{\left(\frac{(P_{0,hot} + \delta_h) - (P_{0,cold} + \delta_c)}{10}\right)} \quad (7)$$

In order to calculate noise parameters of DUT, we need to know Excess Noise Ratio (ENR) of the noise source used for the measurement. The ENR is related to T_{hot} and T_{cold} by [1]

$$ENR_{dB} = 10\log\left(\frac{T_{hot} - T_{cold}}{T_{cold}}\right) \quad (8)$$

The ENR of well-matched devices are usually about 15 dB, which corresponds to T_{hot} » 10,000K. Noise generators are usually calibrated at the factory, and each noise generator is slightly different. The factory prints the data on the noise generator as frequency versus ENR.

Knowing ENR numbers allows us to calculate T_{hot} values of a particular noise pod for all our frequencies of interest.

$$T_{hot} = T_0\left(1 + 10^{\left(\frac{ENR_{dB}}{10}\right)}\right) = 290\left(1 + 10^{\left(\frac{ENR_{dB}}{10}\right)}\right) \quad (9)$$

When the gain and noise figure of DUT is measured, we must first calibrate the meter. It can be done before we make any measurements by connecting the output of the noise pod directly to the input of the noise figure meter or the spectrum analyzer. The meter then measures its own noise temperature and gain. If spectrum analyzer is used we need to write down $P_{0,hot,cal}$ and $P_{0,cold,cal}$ values for each frequency of interest.

Now we have all the numbers to calculate system noise temperature, T_{sys}; system noise factor, F_{sys}; system noise figure, NF_{sys}; and system gain G_{sys} [1]:

$$T_{sys} = \frac{T_{hot} - Y \cdot T_{cold}}{Y - 1} \quad (10)$$

$$F_{sys} = \frac{T_0 - T_{sys}}{T_0} \quad (11)$$

$$NF = 10\log\left(\frac{T_{hot} - T_{cold}}{T_{cold}}\right) - 10\log(Y-1) = ENR_{dB} - 10\log(Y-1) \quad (12)$$

$$G = \frac{P_{0,hot} - P_{0,cold}}{P_{0,hot,cal} - P_{0,cold,cal}} \quad (13)$$

where:

$P_{0,hot,cal}$ is the noise power measured by the meter when the input noise is at T_{hot} and the output of the noise pod is connected directly to the input of the meter;

$P_{0,cold,cal}$ is the noise power measured by the meter when the input noise is at T_{cold} and the output of the

noise pod is connected directly to the input of the meter;
G_{sys} value and P_0 power are in linear terms.

Analogous to Y-value measurement (Eq. 7), four corresponding correction factors can be determined for measurements of $P_{0,hot}$, $P_{0,cold}$, $P_{0,hot,cal}$ and $P_{o,cold,cal}$, and substituted into Equation 13 in order to obtain the right value of G_{sys}:

$$G_{sys} = \frac{\left(P_{0,hot} + \delta_h\right) - \left(P_{0,cold} + \delta_c\right)}{\left(P_{0,hot,cal} + \delta_{h,cal}\right) - \left(P_{0,cold,cal} + \delta_{c,cal}\right)} \qquad (14)$$

where:

d_h is correction factor for $P_{0,hot}$ measurement.
d_c is correction factor for $P_{0,cold}$ measurement.
$d_{h,cal}$ is correction factor for $P_{0,hot,cal}$ measurement.
$d_{c,cal}$ is correction factor for $P_{0,cold,cal}$ measurement.

Conclusion

The accurate measurement of RF signals power level requires consideration of instrumental noise. The measured level is always larger than actual, and correction must be made in order to obtain right numbers. It can be done with the noise de-embedding technique described above for two major tasks. The first is RF signal power level measurement and the second is the measurement of the noise figure of RF devices. This article explains the theoretical basics and shows how to use the noise de-embedding technique to improve the accuracy of RF measurements.

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Reference

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